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A Discrete-Dynamic Study on the Period of Mira Stars

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Abstract

The period of 60 bright Mira stars are studied from a discrete-dynamic point of view. It is found that Mira stars can be classified into three types, to say, Type 1, Type 2 and Type 3. Only two stars belong to Type 1 and seem to be undergoing some evolutionary process. Type 2 and Type 3 are stars on or after the process of approaching the limit cycle oscillation. Some discussion on a relation between period change and spectral type is also given.

1. Introduction

Late-type pulsating variables are divided into several types: Mira type, semi-regular, irregular and RV Tauri type. Of these, Mira type stars are periodic pulsating variables which are late-type giants and supergiants. Their characteristics include a large amplitude in magnitude variation, from 2.5 to more than 10, and a long period of 100–1000 days.

Mira stars show cycle to cycle variations in magnitude and period. The O–C diagram, which shows the difference between observational and calculated times of maximum light, reveals that the period changes in general among Mira type stars over a sufficiently long time. Formerly it was suggested that period changes of most Mira stars may not be real, but show cumalative observational errors. However, nowadays it is thought that period changes are really produced by some physical process, which are perhaps not clearly understood as yet. Therefore, it is very important to investigate any period changes of Mira stars to find their pulsation characteristics.

Recent theoretical studies have considered the period changes of Mira stars in an evolutionary context. For example, Wood and Zarro (1981) suggested that three Mira type stars, which are undergoing continuous period changes, are in the phase just after the Helium-shell flush.

The dynamics of stellar pulsation is described by a third order differential equation which include terms of dissipation. Recent studies of this kind of equation have succeded in explaining periodic pulsating variables such as classical Cepheids and RR Lyrae stars quite well. Existence of periodical and strange behavior solutions has been extensively studied in other type variables. However, we do not have any fully satisfactory model which describe the features of late-type pulsating variables as yet.

Recently TAKEUCHI (1987) considered a "discrete-dynamic" treatment to investigate the pulsation features of semi-regular and irregular variables. However, this has not yet been discussed on the basis of observational quantities.

In this paper we investigate the period changes of 60 bright Mira stars according to Takeuchi's treatment. From the discrete-dynamic point of view, it reveals that Mira stars are divided into three types.

2. Discrete-Dynamic Expression

Consider a period, P_i, as a interval of successive times of i-th and i+1-th maximum light, which is counted from an arbitrary maximum. If a relationship between P_i and P_{i+1} exists independent on i, it must be investigated to derive the proper pulsation characteristics.

Pulsation of Mira stars are considered with "first-return" maps. A first-return map of each star is obtained to plot successive sets of P_i and P_{i+1} on a P_i-P_{i+1} plane on which the horizontal axis is P_i and the vertical axis is P_{i+1}.

TAKEUCHI (1987) considered successive kinetic energy maxima, T(i), with a discrete-dynamic expression of the form, $T(i+1)=f(T(i))$, where every oscillation results from the preceding oscillation. He also discussed the pulsation of stars with first-return maps. A part of his schematic figures are shown in figure 1.

The limit cycle oscillation is expressed at the crossing point of the first-return map and diagonal. If the gradient $df/dT(i)$ at the crossing point is positive, which is indicated by figure 1. a, the pulsation is stable. On the other hand, negative gradient at the crossing point as shown in figure 1. b indicate three possible cases of pulsation. One indicates that pulsation oscillatory approachs to the limit cycle, another shows multi-periodicity and the other shows chaotic change.

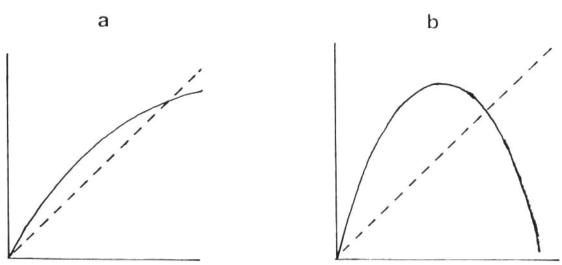

Fig. 1. Schematic first return maps. The horizontal axis indicate T(i) and the vertical axis T(i+1).

From the non-linear theory of stellar oscillation, a linear relation may be derived between the kinetic energy and pulsation period. From this we may describe the relationship between successive periods as,

$$P_i = f(P_{i+1}) \,, \tag{1}$$

which is termed a discrete-dynamic expression.

3. Observational Data

Observational data of times of maximam light of 60 bright Mira stars from 1900 to 1985 have been collected from published and/or unpublished sources. Data from 1900 to 1949 were obtained from the AAVSO (Campbell 1926, Campbell 1955). Data after 1949 are mainly from the JASA (Japan Astronomical Study Association). These data are not complete, especially after 1949, but may suffice to obtain the tendency of first-return maps.

Characteristics of 60 stars obtained from the third (1969) and fourth (1985) editions of the General Catalogue of Variable Stars are tubulated in the first four columns of Table 1. These stars are mainly bright northern Mira stars and observed frequently. Therefore, observational errors of the times of maximam should be small, and seems to be correct within 10 days.

Of these R Aql and R Hya are well known for decreasing periods, and their O–C diagrams show a parabolic form (e.g. DAVIS 1982). WOOD and ZARRO (1981) indicated that period of W Dra increases continuously, and it also shows a parabolic O–C curve. Other stars show period changes to some extent over a long time, and about one third of the stars in Table 1 have shown definite period changes before. For example, Hoeppe (1985) studied the apparent period changes of R Leo over 200 years, and its O–C diagram indicates that R Leo experienced definite period changes several times.

4. Results and Discussion

4.1. *Results*

First-return maps of 60 Mira stars have been obtained and the numerical results are also tabulated in Table 1 after the fifth column. The fifth column of table 1 shows the number of plots obtained. The sixth column shows correlation coefficients for each star. The plots in each figure distribute according to some specific patterns. The type of the pattern for the distribution can be classified into three types. We define these as, Type 1, Type 2 and Type 3, which are indicated in the seventh column. Schematic figures for the three types are illustrated in figure 2.

The description of three types are as follows:

Type 1: Plots distribute linearly on the line of diagonal, figure 2(1), and has rather large value of correlation coefficient.

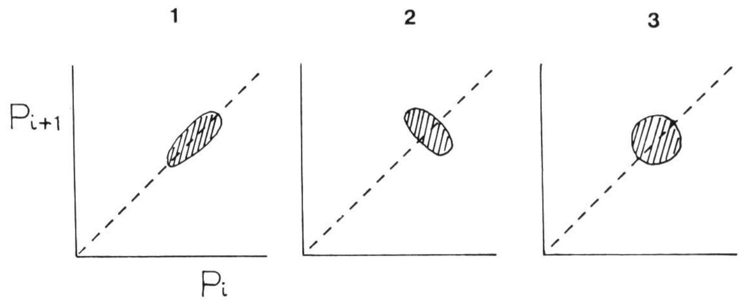

Fig. 2. Schematic figures of three types of distribution of plots, The horizontal axis indicates P_i and the vertical axis P_{i+1}.

Type 2: Plots distribute on the line which cross the line of diagonal and seems to have inverse-correlation, figure 2 (2). Values of correlation coefficients are between −0.35 and −0.6.

Type 3: Plots distribute randomly around a specific point on the diagonal, figure 2(3). Absolute values of correlation coefficients are less than 0.3.

Stars classified as Type 1 consist of only two stars, R Aql and R Hya. The first-return maps of them are shown in figures 3 and 4, respectively. It is noted that their periods are decreasing continuously. R Lep indicates a rather large positive value of correlation coefficient, 0.336. But the distribution of plots of R Lep spread very widely, as shown in figure 5. Therefore, we do not think that R Lep belongs Type 1 but rather Type 3.

The 58 stars excluding R Aql and R Hya are divided into Type 2 and Type 3. 23 stars are Type 2 and 35 stars are Type 3. Plots of these types are shown in figure 6 to 9.

4.2. *Discussion*

The features of the three types of Mira variables are considered as follows:

Type 1: A simple interpretation of this type would be that stars are on the process of approaching the point of limit cycle pulsation on the first-return map, as shown in figure 1. a. However, this idea seems to be incorrect because plots of R Aql and R Hya tend to decline on the diagonal to the origin, as corresponds to decreasing period. Therefore, this type seems to be a result of some specific evolutionary process, as suggested by Wood and Zarro (1981).

Type 2: This seems to be on the process oscillatory approaching a point of limit cycle.

Type 3: This seems to have reached already at a point of limit cycle.

The width of distribution of the three types seems to indicate a variation of activity, including outermost region of star and/or observational errors.

Stars which experienced definite changes of period exist in Type 2 and Type 3.

Table 1. Characteristics and results of Mira stars.

Star	Sp	Amplitude	Period	Number of Plot	Correlation Coefficient	Type
R And	S	5.8–14.9	409.33	60	−0.469	2
W And	S	6.7–14.6	395.93	60	−0.383	2
R Aqr	M	5.8–12.4	386.96	53	−0.125	3
R Aql	M	5.5–12.0	284.2	69	0.665	1
R Ari	M	7.4–13.7	186.78	84	−0.338	2
R Aur	M	6.7–13.9	457.51	46	−0.042	3
R Boo	M	6.2–13.1	223.40	104	−0.374	2
R Cam	S	7.0–14.4	270.22	62	−0.047	3
R Cnc	M	6.1–11.8	361.60	63	0.157	3
R CVn	M	6.5–12.9	328.53	54	−0.010	3
R CMi	C	7.3–11.6	337.78	52	−0.268	3
R Cap	N	9.4–14.9	345.13	27	−0.153	3
R Cas	M	4.7–13.5	430.46	52	−0.362	2
T Cep	M	5.2–11.3	388.14	68	−0.074	3
R Cet	M	7.2–14	166.24	67	−0.396	2
U Cet	M	6.8–13.4	234.76	61	−0.331	2
o Cet	M	2.0–10.1	331.96	69	−0.400	2
R Col	M	7.8–15.0	327.62	30	−0.091	3
R Com	M	7.1–14.6	362.82	46	−0.301	3
S CrB	M	5.8–14.1	360.26	62	−0.066	3
R Crv	M	6.7–14.4	317.03	63	−0.178	3
R Cyg	S	6.1–14.4	426.45	53	−0.610	2
U Cyg	C	5.9–12.1	463.24	43	−0.051	3
RT Cyg	M	6.0–13.1	190.28	116	−0.375	2
α Cyg	S	3.3–14.2	408.05	58	−0.206	3
R Del	M	7.6–13.8	285.07	56	−0.413	3
R Dra	M	6.7–13.2	245.60	93	−0.196	3
W Dra	M	8.9–15.4	278.6	58	−0.470	2
R Gem	S	6.0–14.0	369.91	55	−0.619	2
R Her	M	8.2–15.0	318.14	46	−0.170	3
S Her	M	6.4–13.8	307.28	59	−0.258	3
T Her	M	6.8–13.7	164.98	119	−0.312	3
RS Her	M	7.0–13.0	219.70	64	0.092	3
R Hya	M	3.5–10.9	388.87	68	0.382	1
R Lac	M	8.5–14.8	299.86	50	−0.408	2
R Leo	M	4.4–11.3	309.95	65	−0.132	3
R LMi	M	6.3–13.2	372.19	56	−0.397	2
R Lep	C	5.5–11.7	427.07	44	0.336	3
R Lib	M	9.8–15.9	241.85	23	−0.299	3
RS Lib	M	7.0–13.0	217.65	76	−0.115	2
R Lyn	S	7.2–14.3	378.75	57	−0.519	2
V Mon	M	6.0–13.9	340.5	64	−0.216	3
R Oph	M	7.0–13.8	306.5	67	−0.040	3
X Oph	M	5.9–9.2	328.85	54	0.157	3
U Ori	M	4.8–13.0	368.3	63	−0.104	3
R Peg	M	7.1–13.8	377.84	56	−0.264	3

Table 1. (Continued).

Star	Sp	Amplitude	Period	Number pf Plot	Correlation Coefficient	Type
R Per	M	8.1–14.8	210.0	76	−0.161	3
R Psc	M	7.1–14.8	344.04	53	−0.256	2
R Sgr	M	6.7–12.8	268.6	66	0.005	3
R Sco	M	9.8–15.5	222.83	53	−0.313	2
R Ser	M	5.7–14.4	356.75	63	−0.240	2
R Tau	M	8.2–14.7	324.34	42	−0.462	2
R Tri	M	5.5–12.6	266.40	81	−0.104	3
R UMa	M	6.7–13.4	301.84	78	−0.291	2
S UMa	S	7.4–12.3	225.89	69	−0.381	3
T UMa	M	6.6–13.4	256.88	90	−0.384	3
R Vir	M	6.2–12.1	145.51	125	−0.363	2
S Vir	M	6.3–13.2	377.88	58	−0.357	3
SS Vir	C	6.0–9.6	354.66	51	0.014	3
R Vul	M	7.4–13.4	136.82	105	−0.090	3

Values of characteristics after R Peg are from third edition of GCVS. Others are from fourth edition.

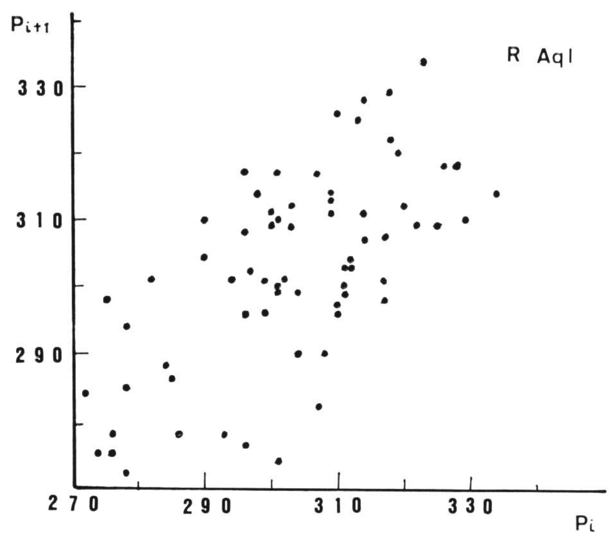

Fig. 3. First return map of R Aql. Large dot shows two dots are at the same place. Hereinafter large dots in figures indicate the same meaning.

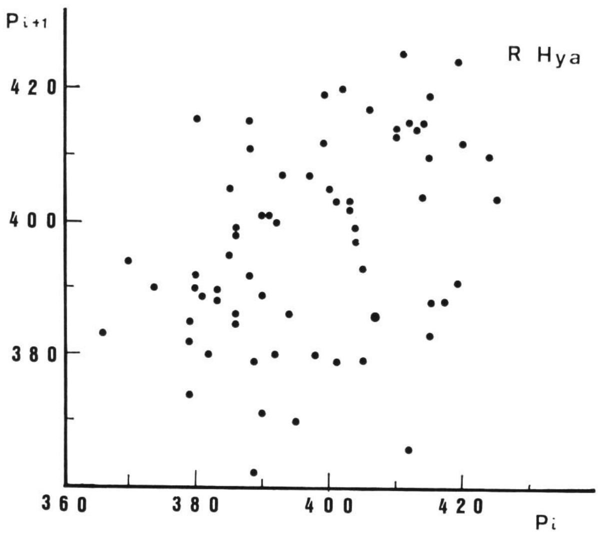

Fig. 4. First return map of R Hya.

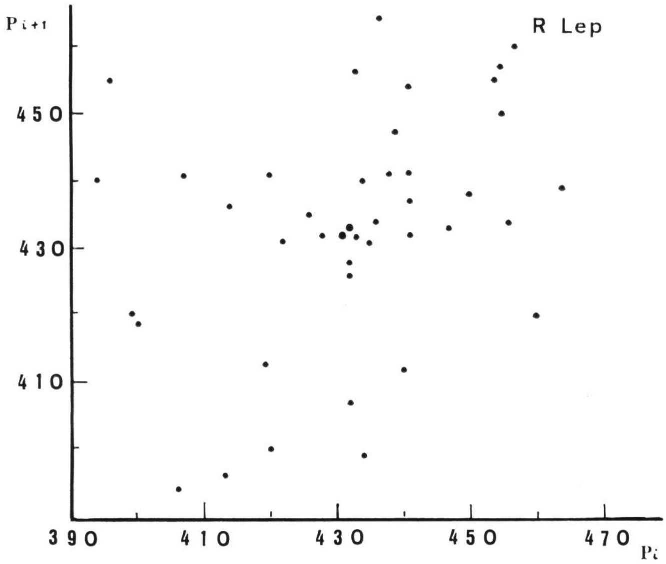

Fig. 5. First return map of R Lep.

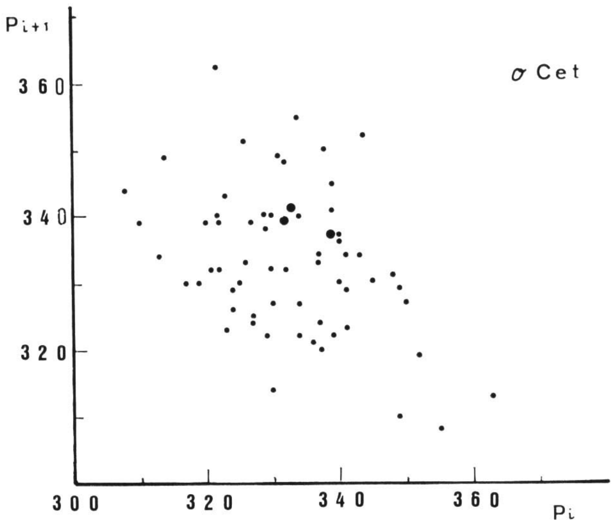

Fig. 6. First return map of *o* Cet.

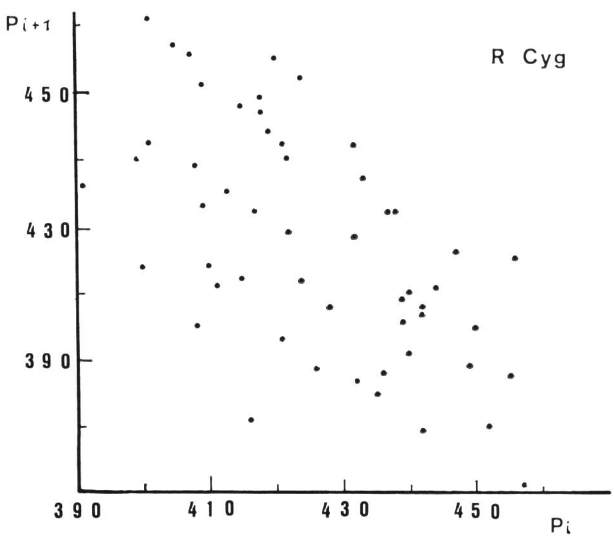

Fig. 7. First return map of R Cyg.

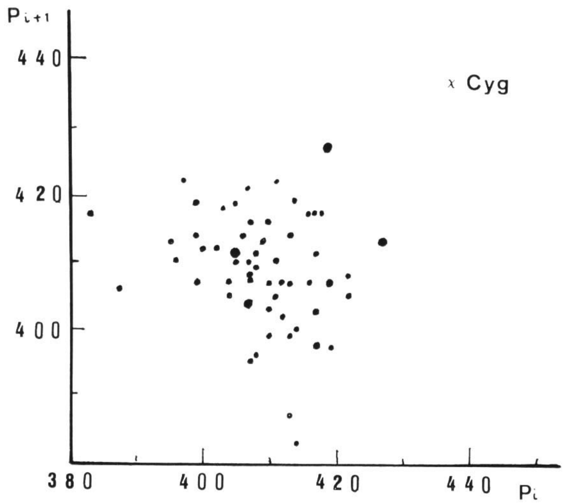

Fig. 8. First return map of χ Cyg.

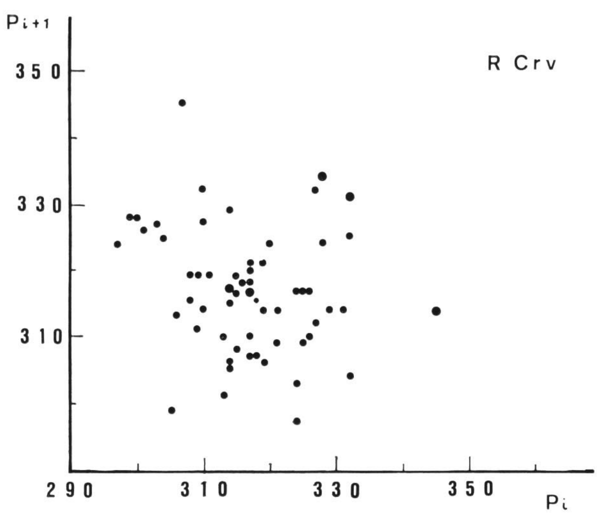

Fig. 9. First return map of R Crv.

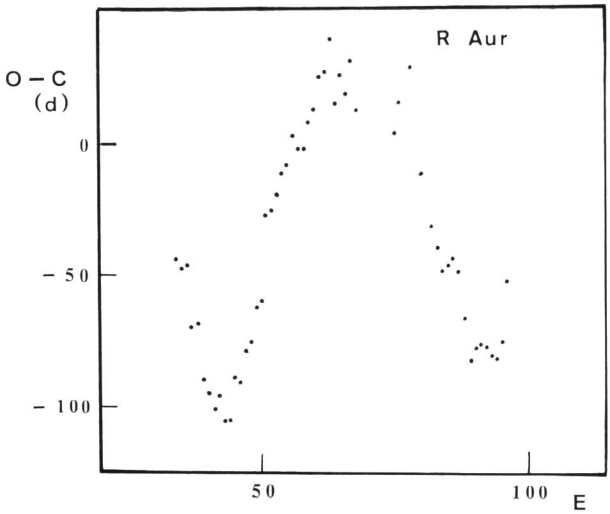

Fig. 10. O-C diagram of R Aur. Horizontal axis indicates thec ycle number, E, from AAVSO. The vertical axis shows O-C in days.

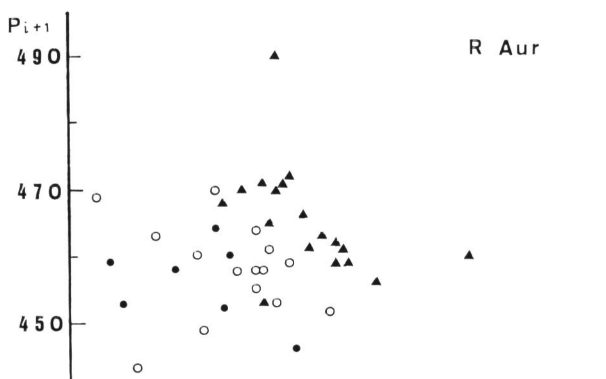

Fig. 11. First return maps of R Aur. Dots indicate before cycle 44. Triangles indicate between cycle 44 and 63. Circles indicate after cycle 63.

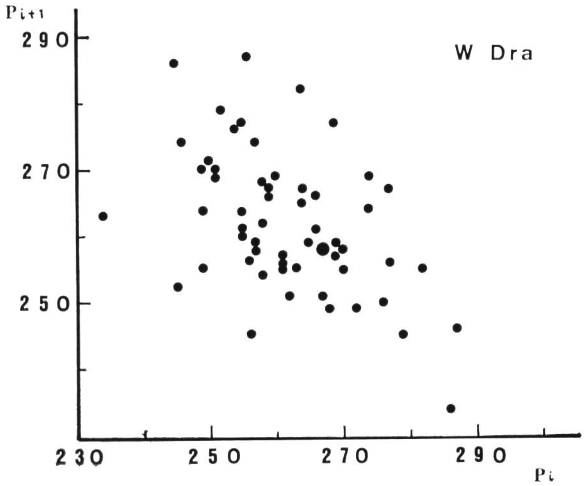

Fig. 12. First return map of W Dra.

Table 2. Distribution of Spectral types.

Sp	Type	Number of Stars	Sp	Type	Number of Stars	Sp	Type	Number of Stars
M	1	2	S	1	0	C(N)	1	0
	2	18		2	5		2	0
	3	27		3	3		3	5

R Aur is one such stars of Type 3. The O–C diagram of R Aur is shown in figure 10. In figure 10 definite changes of period occured at cycle number 44 and 63, where the cycle number has been obtained from AAVSO, in these 80 years. The period after each successive period change is nearly constant. Figure 11 shows plots according to three intervals divided by sudden changes of period. From figure 11 it seems that Type 3 feature is a result of gathering Type 2 like features of each interval. However, this is not clear in the cases of other stars experienced period changes.

The period of W Dra is continuously increasing and shows a parabolic O–C curve (Wood and Zarro 1981). It is noted that W Dra has a Type 2 character, as shown in figure 12, but is different from the Type 1 behavior of R Aql and R Hya, which also show parabolic O–C curves.

Table 2 gives the distribution of 60 Mira stars divided by spectral type. It is noted that all the stars of C (N) type spectrum are Type 3. On the contrary, Type 2 has a rather large proportion of S type stars.

5. Conclusion

Periods of 60 Mira stars have been investigated from the discrete-dynamic point of view to obtain a deeper knoeledge of the character of their pulsation. It is found that Mira stars are classified into three types, Type 1, Type 2 and Type 3. Type 2 and Type 3 stars seem to be on or after the process of approaching the limit cycle point. Some rather special case of Type 1 seems to be undergoing some evolutionary process.

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