Bull. Natn. Sci. Mus., Tokyo, Ser. E, **14**, December 22, 1991

On the Return Map of Mira and Semiregular Carbon Stars

By

Keiichi SAIJO

Department of Science and Engineering, National Science Museum, Ueno Park, Tokyo 110, Japan

Abstract

Peculiar feature of carbon stars is found by return-map study of Mira and semiregular variable stars. Return-maps for period and maximum magnitude are made for 26 carbon Mira and semiregular stars and distribution of types of period-magnitude return-map compared with those of 63 M type stars and 28 S type stars according to our classification of return-maps (SAIJO and WATANABE, 1987; SAIJO, 1988; SAIJO, 1989), Type 1, Type 2 and Type 3. Twenty of 26 carbon stars are found to belong Type 3–1, although M and S type stars concentrate Type 2–3 and Type 3–3.

1. Introduction

It is well known that Mira and semiregular variable stars show irregular behaviour in their light curves to some extent. Although recent development of theoretical studies, which succeed in making models of irregular variability of stellar oscillation, we do not have fully satisfactory model which describe the irregularities of late type pulsating variables as yet. The regular periodic pulsations can be modelled by linear theory but irregular phenomena require non-linear or stochastic theory.

Thus, in order to study the underlying pulsation dynamics of these stars, it is very useful to analyze observational data and to compare them with theoretical calculations. In the course of theoretical consideration, TAKEUCHI (1987) suggested that return maps were very useful to compare theory and observation. SAITOU *et. al.* (1989) found that the complexity of return maps develops according as the decrease of the effective temperature both models and observations of about 30 RV Tauri and semiregular stars.

We have been investigating the irregularity shown in Mira and semiregular variables by using observed return maps. In our previous works (SAIJO and WATANABE 1987, hereinafter referred to as paper I; SAIJO 1988, paper II; and SAIJO 1989, paper III), we classified the pattern of return map into three types, Type 1, Type 2 and Type 3. In paper I, period change of 60 bright Mira stars were analyzed. In paper II, that of 19 Mira stars, whose spectral type are C, were also analyzed to obtain the difference by spectral type of stars. In paper III, change of maximum magnitude in addition to that of period of 21 semiregular stars are analyzed.

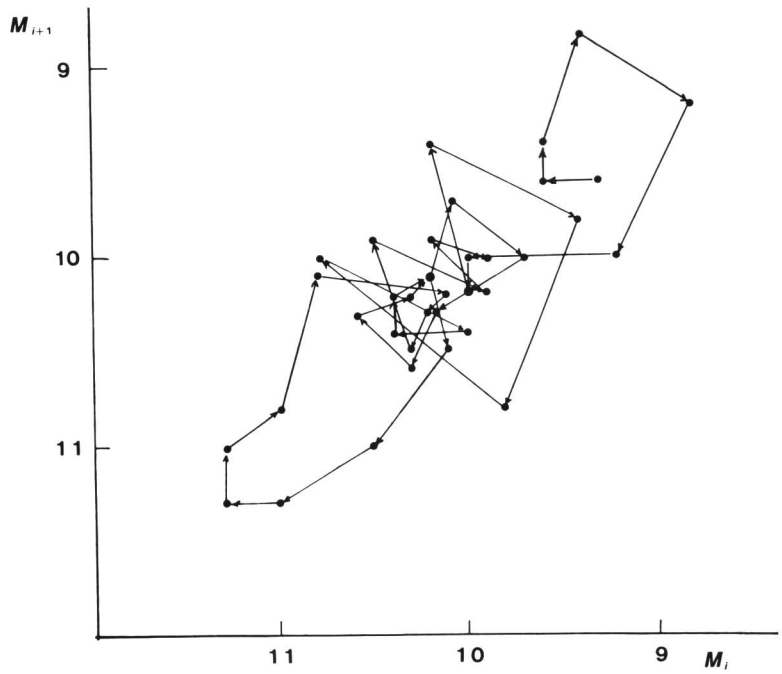

Fig. 1. Magnitude return map of X Cas. The arrowed lines show successive plots. larger dots show two and/or three dost in the same place.

In this paper, we reanalyze the change of maximum magnitude for stars in paper I and paper II and in addition to these stars 20 S-type Mira stars are analyzed to obtain the differences among spectral types, M, S and C. We also report the distribution of return-map type of period-magnitude and found that carbon stars have peculiar distribution.

2. Observational data

For stars in paper I and paper II, observational data of maximum magnitude have been collected from the same sources for paper I to III, that is, from CAMPBELL (1926) and CAMPBELL (1955). For comparison among spectral types, M, S and C, we add 20 S-type Mira stars on CAMPBELL (1926) and CAMPBELL (1955), whose characteristics are shown in table 1 after fourth edition of General catalogue of variable stars (KHOLOPOV 1985).

Thus, we analyze 99 Mira stars, of which 48 stars have M-type spectrum, 28 have S-type and 23 have C-type, and 21 semiregular stars in paper III, of which 15 stars have M-type, 3 stars have C-type and 3 are others (one G-type, Z Aur; two K-type, T Cen and RS Lac).

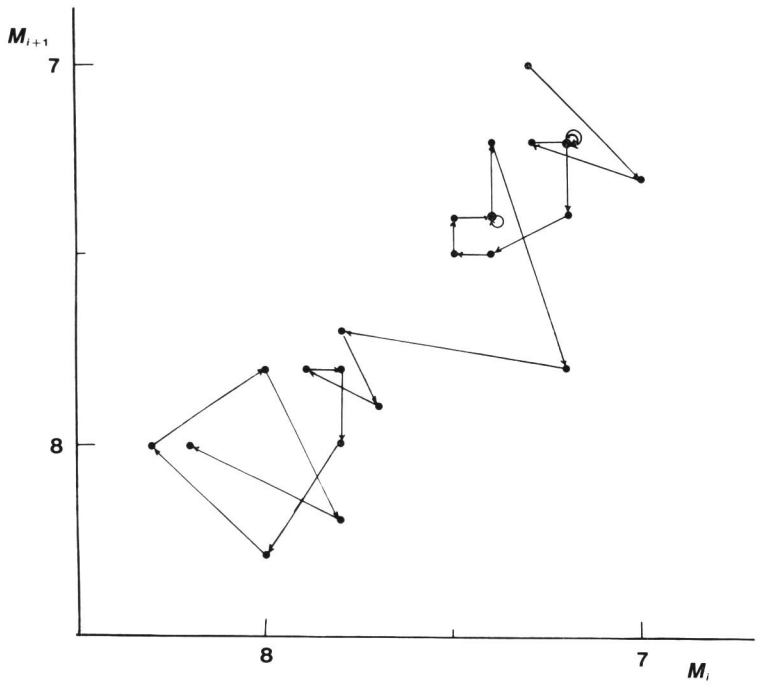

Fig. 2. Magnitude return map of RV Cen.

Table 1. List of additional 20 S-type Mira stars.

Star	Amplitude	Period (Day)
X And	8.3–15.2	346.18
RR Wnd	8.4–15.6	328.15
X Aqr	7.5–14.8	311.65
W Aql	7.3–14.3	490.43
T Cam	7.3–14.4	373.20
V Cnc	7.5–13.9	272.13
RY Car	12.5–<17	424.3
S Cas	7.9–16.1	612.43
U Cas	8.0–15.7	277.19
W Cet	7.1–14.8	351.31
S Cyg	9.3–16.0	322.93
Z Del	8.3–15.3	304.48
T Gem	8.0–15.0	287.79
S Lup	7.8–13.5	339.73
RR Mon	8.4–15.8	394.70
RZ Per	8.7–14.0	355.25
T Sgr	7.1–12.9	394.66
ST Sgr	7.2–16.0	395.12
RT Sco	7.2–15.2	449.04
Z Tau	9.2–14.2	466.2

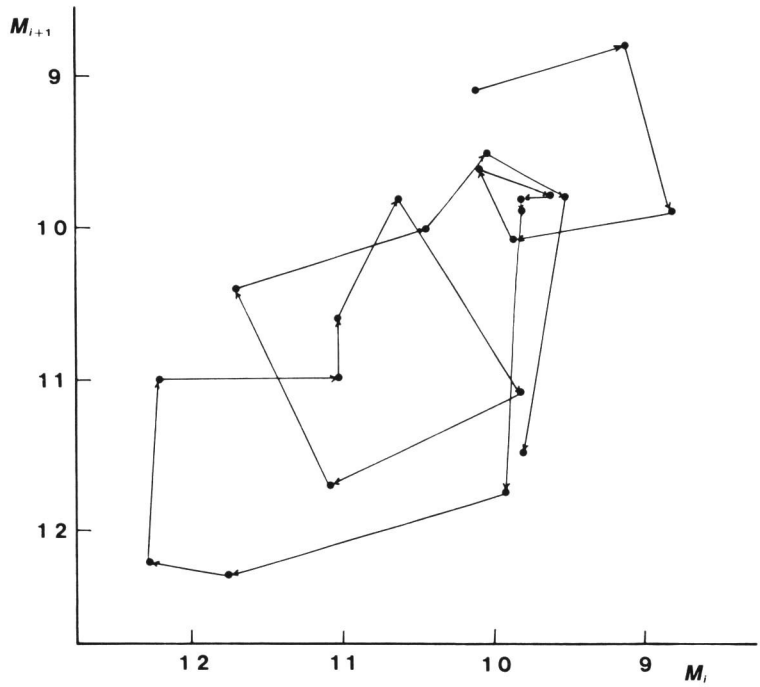

Fig. 3. Magnitude return map of R Vol.

RT Oph in paper II is considered to be a M-type star.

3. Results and discussions

3.1 Results

Table 2 shows characteristics of magnitude return-maps of M-type Mira stars and table 3 shows that of C-type Mira stars. In table 4, characteristics of period and magnitude return-maps of S-type Mira stars are tabulated.

Return maps are classified into three types according to the values of correlation coefficient (SAIJO and WATANABE 1987). Type 1 has a positive value larger than 0.35. Type 2 has a negative value smaller than -0.35. Absolute value of Type 3 is smaller than 0.35, which means no remarkable correlation. Detailed expressions are shown in SAIJO and WATANABE (1987).

Together with results in table 2, 3 and 4 and those in paper I to III, distribution of period-magnitude type of return maps for spectral type and variable type of the stars is shown in table 5.

For M-type Mira stars, large proportion of the stars, 35 of 48 belong to Type 3 of magnitude return-map. Therefore, 21 of 48 stars belong to Type 3–3 and 12 belong to Type 2–3. The same tendency is shown in S-type Mira stars, of which 23 of 28

Table 2. Characteristics of magnitude return-maps of M type Mira stars.

Star	Magnitude		
	Number of plot	Correlation coefficient	Type
R Aqr	22	0.523	1
R Aql	49	−0.068	3
R Ari	55	−0.404	2
R Aur	39	0.189	3
R Boo	60	−0.374	2
R Cnc	33	−0.358	2
R CVn	44	0.282	3
R Cas	36	−0.001	3
T Cep	40	0.201	3
M Cet	31	−01250	3
U Cet	21	−0.363	2
0 Cet	39	−0.478	2
R Col	25	−0.482	2
M Com	47	0.081	3
S CrB	43	−0.149	3
R Crv	32	0.052	3
RT Cyg	78	−0.310	3
R Del	41	0.260	3
R Dra	65	−0.408	2
W Dra	51	−0.206	3
R Her	40	−0.218	3
S Her	50	0.450	1
T Her	94	0.001	3
RS Her	52	−0.264	3
R Hya	25	0.210	3
R Lac	38	0.301	3
R Leo	38	−0.009	3
R LMi	28	−0.079	3
R Lib	8	−0.030	3
RS Lib	40	0.411	1
V Mon	28	−0.250	3
R Oph	28	0.080	3
X Oph	39	0.432	1
RT Oph	20	−0.111	3
U Ori	33	−0.227	3
R Peg	38	−0.294	3
R Per	49	−0.183	3
R Psc	28	0.038	3
R Sgr	30	0.067	3
R Sco	30	−0.352	2
R Ser	45	−0.307	3
R Tau	24	0.221	3
R Tri	36	−0.115	3
R UMa	53	−0.279	3
T UMa	61	−0.386	2
R Vir	53	−0.169	3
S Vir	36	−0.321	3
R Vul	80	−0.075	3

Table 3. Characteristics of magnitude return-maps of C type Mira stars.

Star	Magnitude		
	Number of plot	Correlation coefficient	Type
V Aur	32	0.461	1
R CMi	36	0.372	1
R Cap	14	0.752	1
W Cas	38	0.595	1
X Cas	38	0.718	1
RV Cen	23	0.796	1
S Cep	31	0.479	1
V CrB	40	0.554	1
U Cyg	33	0.566	1
V Cyg	31	0.661	1
WX Cyg	24	0.525	1
T Dra	31	0.766	1
R For	15	0.586	1
R Lep	22	0.683	1
S Lyr	24	0.884	1
U Lyr	30	0.611	1
V Oph	34	0.024	3
R Ori	29	0.156	3
RZ Peg	22	0.197	3
Y Per	54	0.077	3
RU Vir	18	0.665	1
SS Vir	33	0.530	1
R Vol	22	0.611	1

stars belong to Type 3 of magnitude return-map, 16 of 28 belong to Type 3–3 and 7 of 28 belong to Type 2–3. On the contrary, C-type Mira stars concentrate to Type 3 of period return-map and Type 1 of magnitude return-map. Twenty two of 23 stars are Type 1 for period and 19 of 23 are Type 3 for magnitude. Therefore, the majority of C-type stars, 18 of 23, belong to Type 3–1.

For semiregular stars, in which we have no S-type stars, type distribution of M and C stars are similar. They concentrate Type 3 for period and divided about equally in Type 1 and Type 3 for magnitude. However, because of the small sample, no definite relation can find.

3.2 Discussions

Recent theoretical studies of model calculations (*e. g.* SAITOU *et al.* 1989) succeeded in making models of chaotic variation for higher temperature variables of RV Tauri type and W Virginis type than Mira.

For actual stars, CANNIZZO *et al.* (1990) analyzed light curves of two Miras, *o* Cet and R Leo, and one semiregular, V Boo, from continuous long-term data of AAVSO (American Association of Variable Star Observers) by searching correlation

Table 4. Characteristics of return maps of S-type Mira stars.

Star	Period			Magnitude		
	Number of plot	Correlation coefficient	Type	Number of plot	Correlation coefficient	Type
R And*	60	−0.469	2	37	−0.116	3
W Wnd*	60	−0.383	2	37	0.318	3
X And	48	−0.325	3	43	−0.265	3
RR Wnd	49	−0.311	3	33	−0.231	3
X Aqr	27	−0.080	3	14	−0.204	3
W Aql	31	−0.055	3	13	0.123	3
R Cam*	62	−0.047	3	58	0.285	3
T Cam	41	−0.447	2	42	0.263	3
V Cnc	54	−0.258	3	24	−0.350	3
RY Car	16	−0.230	3	17	0.052	3
S Cas	24	−0.207	3	26	0.098	3
U Cas	57	−0.366	2	56	−0.028	3
W Cet	36	−0.250	3	21	0.549	1
R Cyg*	53	−0.610	2	35	−0.250	3
S Cyg	42	−0.155	3	32	−0.062	3
X Cyg*	58	−0.206	3	39	0.094	3
Z Del	47	−0.468	2	30	0.275	3
R Gem*	55	−0.619	2	40	−0.635	2
T Gem	51	−0.224	3	29	0.069	3
S Lup	18	−0.071	3	12	0.247	3
R Lyn*	57	−0.519	2	41	−0.246	3
RR Mon	32	−0.132	3	16	−0.448	2
MZ Per	38	−0.333	3	33	0.583	1
T Sgr	40	−0.186	3	21	−0.066	3
ST Sgr	28	0.059	3	16	−0.438	2
MT Sco	16	−0.246	3	6	−0.334	3
Z Tau	24	−0.298	3	11	−0.176	3
S UMa*	69	−0.381	2	69	0.124	3

* Characteristics of period return maps are from paper I.

Table 5. Distribution of Period-Magnitude type for spectral and variable types.

Type	Mira			Semiregular		
Period-Magnitude	M (48)	S (28)	C (23)	M (15)	C (3)	Others (3)
1–1	0	0	0	0	0	1
1–2	0	0	0	0	0	0
1–3	2	0	0	0	0	0
2–1	1	0	1	0	0	0
2–2	5	1	0	0	0	0
2–3	12	7	0	1	0	0
3–1	3	2	18	8	2	1
3–2	4	2	0	0	0	1
3–3	21	16	4	6	1	0

dimension and found no evidence of low-dimensional atteractors. But the number of stars which have continuous long-term light curves are only a few and the same method is difficult to apply simply to other stars. Since the first-return map is considered as the Poincare section, serious inspection of return maps, which are more easily obtained for many stars, is very useful.

As in section 3.1 and table 5, carbon Mira stars show peculiar distribution of return-map types. Figures 1 to 3, for example, show magnitude return maps of three carbon stars, X Cas, RV Cen and R Vol, of Type 1 feature. In these figures successive plots also indicated by arrowed lines. It is interesting that a feature of clock-round moves enlarged in the direction of the diagonal seems to appear in these figures. Low-dimensional chaotic variation shows torus-like feature sometimes.

Carbon stars are widely considered to be formed by dredge-up process of heavy elements from the stellar core to the surface. M-type stars can transfer to carbon stars as changes of the atmosphere from oxigen-rich to carbon-rich. Therefore, the peculiarity of type distribution seems to indicate the effect of stellar evolution.

4. Summary

Variations of period and maximum magnitude of Mira and semiregular variable stars are investigated by using return maps as successive study of our earlier works of paper I to III, in which we introduced three types for classification of return maps.

Together with the results in paper I to III, type distribution of return maps are analyzed for 63 M-type, 28 S-type and 26 carbon stars. It is found that period-magnitude type of carnon Mira stars show peculiar distribution, 18 of 23 stars belong to Type 3–1. On the contrary, M-type and S-type spectrum stars are concentrate in Type 3–3 and type 2–3.

Acknowledgement

This work is partly supported by the Scientific Research Fund of the Ministry of Education, Science, and Culture under Grant No. 02640641.

Refernces

CAMPBELL, L., 1926. Maxima and minima of two hundred and seventy-two long period variable stars. *Annals of the Astronomical Observatory of Harvard College*, Vol. **79**., Part 2.

CAMPBELL, L., 1955. Studies of long period variables. American Association of Variable Star Observers, Cambridge.

CANNIZZO, J. K., GOODINGS, D. A., and MATTEI, J. A., 1990. A search for chaotic behavior in the light curves of three long-period variables. *Astrophy. J.*, **357**: 235.

KHOLOPOV, P. N. (ed.), 1985. General catalogue of variable stars. 4th-ed., Astronomical Council of the USSR Academy of Sciences, Moscow.

SAIJO, K., 1988. On the period of Mira type carbon stars: A discrete-dynamic study. *Bull. Natn.*

Sci. Mus., Ser. E, Vol. **11**., 1.

SAIJO, K., 1989. A return-map study of semiregular variable stars: Period and magnitude. *Bull. Natn. Sci. Mus.*, Ser. E, Vol. **12**, 21.

SAIJO, K. and WATANABE, M., 1987. A discrete-dynamic study on the period of Mira stars. *Bull. Natn. Sci. Mus.*, Ser. E, Vol. **10**., 1.

SAITOU, M., TAKEUCHI, M., and TANAKA, T., 1989. Chaotic behavior in nonlinear radial oscillations of one-zone stellar models. *Publ. Astron. Soc. Japan*, **41**, 297.

TAKEUCHI, M., 1987. A discrete-dynamic study of pulsating stars. *Astrophys. Space Sci.*, **136**: 129.